List of Equations of use for the CERN LIS

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Sources

Child-Langmuir Law [1] p635

$$J = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2q}{m}} \cdot \frac{U_d^{3/2}}{d^2}$$

With initial velocities [2]

$$\theta = \left\{ \left[(\eta+1)^{1/2} - \eta^{1/2} \right]^{3/2} + 3\eta^{1/2} \left[(\eta+1)^{1/2} - \eta^{1/2} \right]^{1/2} \right\}^2$$

Plasma Potential

$$\phi_p = \frac{kT_e}{2e} \ln\left[\frac{M_i T_e}{M_e T_i} \right]$$

Radius of curvature of plasma meniscus

$$R_1 = 1.6d\, P_{D0} / \left(P_{D0} - P_D\right)$$

Debye Length in a Plasma

$$\lambda_D[\mathrm{cm}] = 743 \sqrt{\frac{T_e[\mathrm{eV}]}{n_e[\mathrm{cm}^{-3}]}}$$

Plasma Frequency

$$\omega_p = \sqrt{\frac{e^2 n_e}{\varepsilon_0 m}}$$

Cyclotron frequency [3]p16

$$\omega_c = \frac{eB}{m},\ f_{ci} = 1.52QB/A[\mathrm{MHz}],\ f_{ce} = 2.8B[\mathrm{GHz}]$$ B in kGauss

$$r_c = \frac{v_\perp}{\omega_c} = \frac{\sqrt{2mE_\perp}}{eB} = 3.4\frac{\sqrt{E_\perp[eV]}}{B[T]}\,\mu\mathrm{m}$$ $E_\perp$ is the rotational energy

Inverse Bremstrahlung Absorption

$$k = \frac{4\sqrt{2\pi}Z^2e^4n_i\Lambda_{ei}}{3cT_e^{3/2}m_e^{1/2}}\frac{\left(\frac{n_e}{n_{cr}}\right)^2}{\left(1-\frac{n_e}{n_{cr}}\right)^{1/2}}$$

Lasers

Focal spot for a Gaussian

Emittance

The 4rms emittance [3]p32

$$\varepsilon_{4rms} = \frac{4}{N^2}\left\{\left[N\sum x^2 - \left(\sum x\right)^2\right]\left[N\sum x'^2 - \left(\sum x'\right)^2\right] - \left[N\sum xx' - \sum x \sum x'\right]^2\right\}^{1/2}$$

For a centred beam [5] p52

$\beta\varepsilon=\langle x^2\rangle$; $\gamma\varepsilon=\langle x'^2\rangle$; $\alpha\varepsilon=\langle xx'\rangle$

The emittance ellipse [3]p61.

$$\gamma x^2 + 2\alpha xx' + \beta x'^2 = \varepsilon$$

Relation

$$\beta\gamma - \alpha^2 = 1$$

Phase Space ellipse definitions.

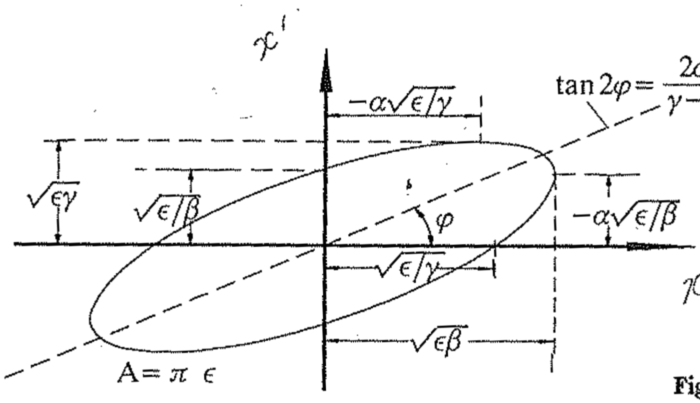

Fig. 5.22. Phase space ellipse

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Normalised emittance [3]p33

$$\varepsilon_n = \beta\gamma\varepsilon$$

Relativity (p=momentum, W=kinetic energy, E_0=rest mass) – Atomic mass = 931.49MeV/c^2

$$cp = \left[W\left(2E_0 + W\right)\right]^{1/2}$$

$$W = E_0\left[\sqrt{1+\left(\frac{cp}{E_0}\right)^2} - 1\right]$$

$$\frac{dP}{P} = \frac{W\left(E_0 + W\right)}{P^2}\frac{dW}{W}$$

$$\left(B\rho\right) = 3.3356\ \left(pc\right)\ \left[\mathrm{GeV}\right]$$

References

[1] Handbook of Ion Sources, Ed B Wolf, CRC Press, 1995.
[2] Extraction of an Ion Beam from a Laser Ion Source, R Scrivens, PhD Thesis, University of Wales, Swansea, 1999.
[3] The Physics and Technology of Ion Sources, Ed I Brown, J Wiley and Sons, 1989.
[4] Trace 3D Documentation, KR Crandall and DP Rusthoi, LA-11054-MS, 1990.
[5] Handbook of Accelerator Physics, Chao and Tigner.