To calculate in 1D the focal distribution of a Gaussian profile, where

$$I = Ce^{-x^2/2\sigma^2} \tag{1}$$

use the transform

$$A(l) = \int f(x)\exp(\frac{-2\pi i}{\lambda} lx)dx \tag{2}$$

where *f(x)* is the wave amplitude and *l* is the directional cosine which we can approximate to θ.

so

$$A(l) = \int C\exp(-x^2/4\sigma^2)\exp(\frac{-2\pi i}{\lambda} lx)dx \tag{3}$$

$$A(l) = \int C\exp(-\left(\frac{x}{2\sigma} + \frac{2\pi i l\sigma}{\lambda}\right)^2)\exp(\frac{-4\pi^2 l^2\sigma^2}{\lambda^2})dx \tag{4}$$

The second exponential is just a constant, while the first should be replaced with exp($-u^2$). Then integration gives

$$A(l) = 2\sigma C\sqrt{\pi}\exp(-\frac{4\pi^2\sigma^2 l^2}{\lambda^2}) \tag{5}$$

If we now square this to get the intensity, we have a Gaussian distribution with

$$I_f = C_2\exp(-l^2/2\sigma_f^2) = C_2\exp(-\frac{16\pi^2\sigma^2 l^2}{2\lambda^2}) \tag{6}$$

so finally we have

$$\sigma_f = \lambda/4\pi\sigma \tag{7}$$

Normally, I use the focal spot $d=4\sigma_f$, and the input beam $D=4\sigma$.

From this, and the conversion to a lens, so $l=d/F$ where F is the lens focal length, we find the formula

$$d_{4\sigma} = 4F\lambda/\pi D_{4\sigma} \tag{8}$$